Exercise 28

The table shows how the average age of first marriage of Japanese women has varied since 1950.

t	$A(t)$	t	$A(t)$
1950	23.0	1985	25.5
1955	23.8	1990	25.9
1960	24.4	1995	26.3
1965	24.5	2000	27.0
1970	24.2	2005	28.0
1975	24.7	2010	28.8
1980	25.2		

(a) Use a graphing calculator or computer to model these data with a fourth-degree polynomial.

(b) Use part (a) to find a model for $A'(t)$.

(c) Estimate the rate of change of marriage age for women in 1990.

(d) Graph the data points and the models for A and A'.

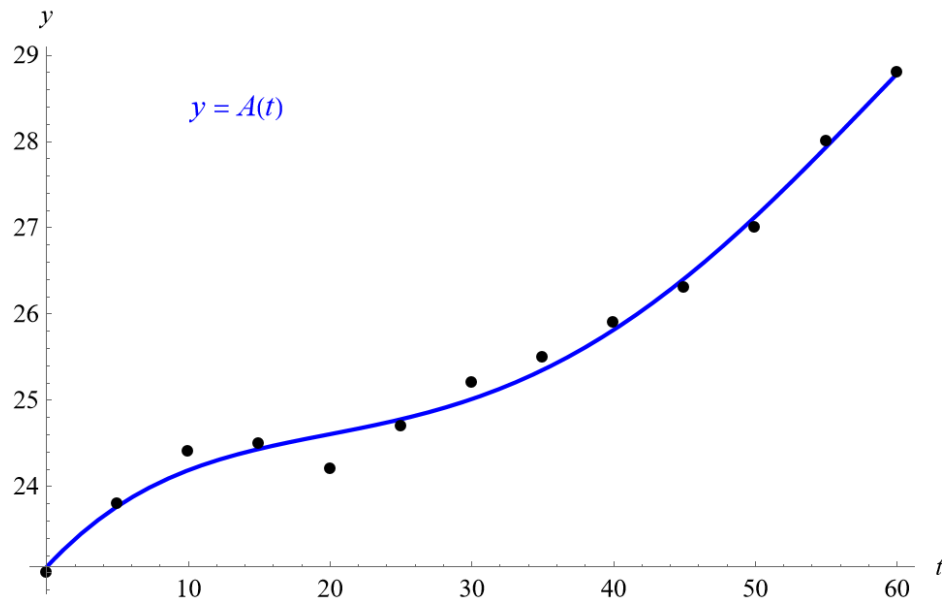
Solution

Part (a)

Let t instead be the number of years after 1950. Mathematica's FindFit function gives

$$A(t) \approx (-1.19978 \times 10^{-6})t^4 + 0.000187563t^3 - 0.00830855t^2 + 0.177721t + 23.0681$$

as the fourth-degree polynomial that best fits the data.



Part (b)

Take the derivative of this function to get the rate that the marriage age increases per year.

$$\frac{dA}{dt} \approx \frac{d}{dt}[(-1.19978 \times 10^{-6})t^4 + 0.000187563t^3 - 0.00830855t^2 + 0.177721t + 23.0681]$$

$$\approx (-1.19978 \times 10^{-6})(4t^3) + 0.000187563(3t^2) - 0.00830855(2t) + 0.177721(1) + 0$$

$$\approx (-4.79912 \times 10^{-6})t^3 + 0.00056269t^2 - 0.0166171t + 0.177721$$

Part (c)

Plug in $t = 40$ to get the rate of change of the marriage age per year in 1990.

$$\frac{dA}{dt}(40) \approx (-4.79912 \times 10^{-6})(40)^3 + 0.00056269(40)^2 - 0.0166171(40) + 0.177721$$

$$\approx 0.106198$$

Part (d)

Below is a graph of the model for $A'(t)$ versus t.